MECHANISM FOR COALESCENCE OF GAS BUBBLES MOVING IN A VISCOUS FLUID

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When investigating processes which take place in a fluid containing gas- or vapor bubbles, it is very important to understand the mechanism of bubble growth during bubble motion in the fluid. Evidently, the most significant process here is that of coalescence, that is, the convergence of fine drops and their combination to form a drop of larger dimension. In their studies of the dynamic mechanism of coagulation and coalescence, [1-3] examined the macroscopic motion of heterogeneous mixtures, which usually satisfied a number of assumptions. These assumptions, as a rule, make it possible to investigate the dynamic characteristics of individual particles with subsequent attempts to account for pairwise interactions between particles. So, the Berkness force, which causes particles to converge, is considered as the result of the interaction of two distinct particles. At the same time, there exists another possibility for the interaction of particles and their collective dynamics, which is related to the coherent character of long-wave disturbances. A large number of particles participate directly in the formulation of these disturbances. The possibility was first pointed out in [4], where, however, the use of a potential flow scheme precluded generalization of the results to the case of real fluids. In [5, 6], viscosity was taken into account in the study of the collective interaction in problems of this nature, when investigating the flow of aerosols, and in the study of the interaction of sound in an aerosol. It was shown that in a gas containing moving solid particles, instabilities developed which resulted in the convergence of particles and their subsequent coagulation. The instability has a simple explanation. If in some region, there is an increase in the volumetric concentration of particles, then, as follows from the continuity equation, there is also an increase in the velocity of the carrier phase in this region. As a result, particles from the region of increased concentration are accelerated and catch up with particles in the region of lower concentration. This leads to subsequent growth in the volumetric density and an increase in the probability of coagulation. In addition, with growing gas velocity in the region of increased particle concentration, the pressure is reduced, which causes a further increase in the particle number density.

Since this instability is of a universal character, it becomes obvious that it can arise during any smooth flow in a two-phase medium. It is of great interest to study this instability in a fluid with gas- or vapor bubbles. This is because it is necessary to have a more complete understanding of the processes which take place during fluid degassing [2]. In addition, a characteristic oscillation arises in the bubbles in such a medium during the propagation of disturbances. Accounting for this can in one way or another influence the development of the process. Our work is devoted to a solution of this problem.

We carry out our examination using the example of a simple model of a viscous fluid which contains bubbles of gas of one radius, which drift to the fluid surface in a gravity field. We assume that energy and other effects of chaotic and internal motion of the bubbles can be ignored. We will assume that bubble collision, fragmentation, and agglutination do not occur, and that phase transitions also do not take place. We consider only the radial oscillations of the drops. Then the system of equations for a similar model of a two-phase medium has the form [7]

$$\operatorname{div} \mathbf{v}_1 = \partial\alpha/\partial t + \operatorname{div} \alpha\mathbf{v}_1; \tag{1}$$

$$\partial n/\partial t + \operatorname{div} n\mathbf{v}_2 = 0; \tag{2}$$

$$d_2\rho_2/dt = -(3\rho_2/a)d_2a/dt; \tag{3}$$

$$\rho_1 \frac{d_1\mathbf{v}_1}{dt} = -\nabla p^* + \nabla\tau^* - \frac{\alpha}{2}\rho_1\left[\frac{d_2}{dt}(\mathbf{v}_1-\mathbf{v}_2) - \frac{3}{a}(\mathbf{v}_2-\mathbf{v}_1)\frac{d_2a}{dt}\right] - \frac{9\alpha}{2a^2}\nu\rho_1(\mathbf{v}_1-\mathbf{v}_2) + \rho_1\mathbf{g}; \tag{4}$$

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$$\rho_2 \frac{d_2 \mathbf{v}_2}{dt} = \rho_1 \left(\frac{d_1 \mathbf{v}_1}{dt} - \mathbf{g} \right) + \frac{\rho_1}{2} \left[\frac{d_2}{dt} (\mathbf{v}_1 - \mathbf{v}_2) - \frac{3}{a} (\mathbf{v}_2 - \mathbf{v}_1) \frac{d_2 a}{dt} \right] + \frac{9}{2a^2} \nu\rho_1 (\mathbf{v}_1 - \mathbf{v}_2) + \rho_2 \mathbf{g}; \tag{5}$$

$$a d_2^2 a/dt^2 = (p_2 - p_1 - 2\sigma/a)/\rho_1 - (4\nu/a)\, d_2 a/dt - (3/2)(d_2 a/dt)^2 + (\mathbf{v}_1 - \mathbf{v}_2)^2/4; \tag{6}$$

$$p^* = (1 - \alpha) p_1 + \alpha(p_2 - 2\sigma/a) + \alpha\rho_1 [(d_2 a/dt)^2 + (\mathbf{v}_2 - \mathbf{v}_1)^2/6]; \tag{7}$$

$$\tau^* = -\alpha\rho_1 (\mathbf{v}_2 - \mathbf{v}_1)^2/3 + 4\nu\rho_1 \nabla [(1 - \alpha)\mathbf{v}_1 + \alpha\mathbf{v}_2]/3. \tag{8}$$

Here, (1)-(3) are the equations of conservation of specific volumes of the phases and of particle number; (4) and (5) the equations of motion of the fluid and the gas, respectively; (6) is the Rayleigh–Lamb equation for bubble oscillation; $\mathbf{v}_1$ and $\mathbf{v}_2$ the velocities of the fluid and bubbles; ρ_1, ρ_2 the fluid and gas densities; a the bubble radius; $\alpha = 4\pi a^3 n/3$ the volumetric concentration of bubbles; ν the kinematic viscosity of the fluid; $d_1/dt = \partial/\partial t + \mathbf{v}_1\nabla$; $d_2/dt = \partial/\partial t + \mathbf{v}_2\nabla$; p_1 the pressure in the fluid; p_2 that of the gas in the bubbles, which from the equation of state can be determined in the form $p_2 = p_0(a_0/a)^{3\gamma}$; γ is the adiabatic index; and p the pressure of the gas in a bubble of radius a_0.

It is easy to see that (1)-(8) has a steady solution, corresponding to uniform drift of the bubbles in the gravity field. In general, such a solution depends on the coordinates of the drifting bubble, that is, the parameters of the bubble slowly change according to the degree of drift. However, it is obvious that a change in the parameters can be neglected over a scale length $h \ll p_0/\rho_1 g$, where p_0 is the pressure in the fluid taken in equilibrium (for example, if for p_0, we take the pressure at the fluid surface, then $h \ll 10$ m). In addition, (1)-(8) are valid for the assumption of small Reynolds numbers (Re < 1), and also under the condition that

$$a \ll n^{-1/3} \ll \lambda \tag{9}$$

(λ is the characteristic scale of hydrodynamic disturbances).

To investigate the instability, we switch over to a reference frame fixed to the bubbles, and considering the magnitudes p_0, v_{10}, n_0, a_0 as unperturbed quantities, we linearize (1)-(8) in terms of these quantities. By assuming that the disturbance depends on the coordinates and on time according to $\exp(-i\omega t + i\mathbf{k}\mathbf{r})$, we find

$$\mathbf{k}\mathbf{v}_1' = -\alpha'(\omega - \mathbf{k}\mathbf{v}_{10}); \tag{10}$$

$$\mathbf{k}\mathbf{v}_2' = \omega(\alpha'/\alpha - 3a'/a); \tag{11}$$

$$\rho_2' = -3\rho_2 a'/a; \tag{12}$$

$$p_1'/\rho_1 = a'(a_0\omega^2 + 4i\nu\omega/a_0^2 - 3\gamma p_0/a\rho_1) + \mathbf{v}_{10}(\mathbf{v}_1' - \mathbf{v}_2')/2; \tag{13}$$

$$\mathbf{k}\mathbf{v}_2'(\omega/2 + i\beta) = \mathbf{k}\mathbf{v}_1'(3\omega/2 - \mathbf{k}\mathbf{v}_{10} + i\beta) + a'(3\omega/2 - 2i\beta)\mathbf{k}\mathbf{v}_{10}/a; \tag{14}$$

$$k^2 p_1'/\rho_1 = \mathbf{k}\mathbf{v}_1'(\omega - \mathbf{k}\mathbf{v}_{10} + 4i\nu k^2/3 + i\beta\alpha) + \alpha\mathbf{k}\mathbf{v}_2'(\mathbf{k}\mathbf{v}_{10} + 4i\nu k^2/3 - \omega/2 - i\beta) + \alpha'[-(\mathbf{k}\mathbf{v}_{10})^2/2 - (4i\nu k^2/3)\mathbf{k}\mathbf{v}_{10} + i\beta\mathbf{k}\mathbf{v}_{10}] + a'\alpha\mathbf{k}\mathbf{v}_{10}(3\omega/2 - 4\beta)/a, \tag{15}$$

where $\beta = 9\nu/2a_0^2$.

Solving (10)-(15) for oscillations in the system, we obtain the dispersion equation

$$[-3\alpha\omega^2/2 + \omega(\varkappa\Omega/2 - i\beta\alpha) - \alpha\varkappa\Omega(2\varkappa\Omega - i\beta)][3\omega^2/2 + \omega(3\varkappa\Omega/2 + 3i\beta) - 2i\beta\varkappa\Omega] = [\omega^2(\varkappa^2 - 3\alpha/2) + \omega(3\varkappa\Omega/2 - 3i\beta\alpha + 4i\beta\varkappa^2/3) + (2i\beta\alpha\varkappa\Omega - \varkappa^2\omega_0^2)][\omega^2/2 + \omega(i\beta - 5\alpha\varkappa\Omega/2) + \alpha\varkappa\Omega(\varkappa\Omega - i\beta)] \tag{16}$$

using the relations $\alpha \ll 1$, $\rho_2/\rho_1 \ll 1$, $2\sigma/a \ll p_0$. In addition, we have introduced the notation: $\omega_0 = (c/a)\sqrt{\rho_2/\rho_1}$, $\Omega = v_{10}/a_0$, $\kappa = ka_0$, c is the velocity of sound in the gas. Further transforming (16) for the relation of the frequencies with the assumptions $\omega_0 \gg \beta$, $\omega_0 \gg \alpha$, we have

$$\omega^4 + (10i\beta/3 - 5\alpha\varkappa\Omega)\,\omega^3 - \left(\omega_0^2 + 26i\beta\frac{\alpha}{3}\varkappa\Omega\right)\omega^2 + \left[-2i\beta\omega_0^2 + (6\beta^2 + 5(\varkappa\omega_0)^2)\,\alpha\Omega/\varkappa\right]\omega - -2\alpha\Omega^2((\varkappa\omega_0)^2 + 2\beta^2) + 2i\beta\alpha\varkappa\Omega\omega_0^2 = 0.$$

The roots of the dispersion equation are divided into high-frequency values with $\omega \sim \omega_0$ and low-frequency values. The solution shows that in the high-frequency approximation, the oscillations are stable and are of no interest. In the low-frequency approximation ($\omega \ll \omega_0$, β), only the last two terms are significant, which gives

$$\omega \simeq \alpha\varkappa\Omega + i\alpha\Omega^2(\varkappa^2 + 2\beta^2/\omega_0^2)/\beta. \tag{17}$$

It is clear from (17) that in the two-phase medium considered here, there develops an instability whose increment is

$$\gamma \approx \alpha\Omega^2(\varkappa^2 + 2\beta^2/\omega_0^2)/\beta.$$

The instability leads to growth in the disturbance of the concentration and convergence of the bubbles. This in turn increases the probability of bubble coalescence. Clearly, for oscillations with $\kappa \ll \beta/\omega_0$, the solution agrees with that obtained in [5, 6]. If $\kappa \gtrsim \beta/\omega_0$, the characteristic oscillations of the bubbles must be taken into account. Quantitative estimates indicate that for the parameters $a_0 \sim 0.01$ cm, $\alpha \sim 0.02$, $v = 2ga^2/9\nu$, then $\gamma \sim 0.1\ \text{sec}^{-1}$. Thus, the characteristic time for development of the instability, that is, the time for bubble convergence, is $\tau \sim 10$ sec. In this time period, the bubbles transverse a distance $\ell = 20$ cm, which makes it possible to ignore smooth changes in the unperturbed values and underscores the validity of the simplifications made here. Note that the described mechanism can explain not only the coalescence of bubbles drifting in the fluid, but also coalescence during the action of powerful acoustic radiation on the fluid [2].

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